COURSE DETAILS

Credits : 4
Course Category : PE
Pre-Requisites (s) : Mechanical Vibrations
Contact hours : 3-0-1
Type of Course : Theory
Course Assessment : Continuous Evaluation 15%, Mid-Semester Examination 25%
End-Semester Examination 60%

Course Objectives:

• Impart basic understanding of the rotor dynamics phenomena with the help of simple rotor models and subsequently carry out the analysis for real life rotor systems.
• Ability to write down the differential equations of motion for simple, geared and branched rotor bearing system under transverse and torsional vibrations.
• Capability to find out the critical speeds using different numerical methods, balance the unbalanced system and perform the instability analysis.
• Apply of the knowledge of mathematics, science and engineering for the analysis and design of rotor-shaft systems with different kinds of bearings.
• To be capable to boost research in the developing area of the rotor dynamics such as identification of rotor bearing system parameters and its use in futuristic model based condition monitoring and fault diagnostic.
COURSE DETAILS

Course Outcomes

• Proficiency to analyze the various effects associated with the rotor dynamics.
• Ability to develop the vibration models of rotor bearing systems with changing complexities for real engineering systems.
• Ability to formulate the response due to unbalance and instability in practical rotor systems.
• Ability to use various vibration measuring and balancing instruments.
• Ability to identify rotor bearing system parameters and capability to carry out research in condition monitoring and fault identification in rotors.

Syllabus:

• Introduction, Simple rotors with rigid bearings, Jeffcott rotor model and variant of Jeffcott rotor model, Shafts stiffness constants, Rotor-bearing interactions: Effects of rolling element bearings and fluid film bearings on rigid and flexible rotors.
• Flexural and torsional vibrations; critical speeds of shafts using Rayleigh’s method, matrix iteration methods, Prohal and Myklested method; equivalent discrete systems; geared and branched systems; Gyroscopic effects.
• Instability of rotors mounted on fluid film bearings; rigid rotor instability; instability of a flexible rotor; instability threshold by transfer matrix methods; internal hysteresis of shafts; instability in torsional vibrations.
• Balancing of rotors and balancing criteria for rigid and flexible rotors; bearing dynamic parameters estimation; measurement & digital processing techniques; condition monitoring of rotating machineries.
COURSE DETAILS

Books:

<table>
<thead>
<tr>
<th>Course Outcomes</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>i</th>
<th>j</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>L</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>M</td>
<td>L</td>
<td>M</td>
<td>M</td>
<td></td>
<td></td>
<td>L</td>
<td>L</td>
<td>M</td>
</tr>
<tr>
<td>3</td>
<td>L</td>
<td></td>
<td></td>
<td>M</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>H</td>
<td>M</td>
<td></td>
<td></td>
<td>M</td>
<td></td>
<td>L</td>
</tr>
<tr>
<td>5</td>
<td>H</td>
<td>L</td>
<td>M</td>
<td>M</td>
<td>L</td>
<td></td>
<td></td>
<td>L</td>
<td></td>
<td>M</td>
</tr>
</tbody>
</table>
# Tentative Teaching Schedule

<table>
<thead>
<tr>
<th>Topic</th>
<th>No. of Lectures</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNIT I</strong></td>
<td></td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Simple rotors with rigid bearings</td>
<td>2</td>
</tr>
<tr>
<td>Jeffcott rotor model and variant of Jeffcott rotor model</td>
<td>4</td>
</tr>
<tr>
<td>Shafts stiffness constants</td>
<td>1</td>
</tr>
<tr>
<td>Rotor-bearing interactions: Effects of rolling element bearings and fluid film bearings on rigid and flexible rotors</td>
<td>5</td>
</tr>
<tr>
<td><strong>UNIT II</strong></td>
<td></td>
</tr>
<tr>
<td>Flexural and torsional vibrations</td>
<td>5</td>
</tr>
<tr>
<td>Critical speeds of shafts using Rayleigh’s method, Matrix iteration method, Probal and Myklest method</td>
<td>2</td>
</tr>
<tr>
<td>Equivalent discrete systems</td>
<td>2</td>
</tr>
<tr>
<td>Geared and branched systems</td>
<td>2</td>
</tr>
<tr>
<td>Gyroscopic effects</td>
<td>3</td>
</tr>
<tr>
<td><strong>UNIT III</strong></td>
<td></td>
</tr>
<tr>
<td>Instability of rotors mounted on fluid film bearings</td>
<td>2</td>
</tr>
<tr>
<td>Rigid rotor instability</td>
<td>3</td>
</tr>
<tr>
<td>Instability of a flexible rotor</td>
<td>3</td>
</tr>
<tr>
<td>Instability threshold by transfer matrix methods</td>
<td>2</td>
</tr>
<tr>
<td>Internal hysteresis of shafts</td>
<td>1</td>
</tr>
<tr>
<td>Instability in torsional vibrations</td>
<td>2</td>
</tr>
<tr>
<td><strong>UNIT IV</strong></td>
<td></td>
</tr>
<tr>
<td>Balancing of rotors and balancing criteria for rigid and flexible rotors</td>
<td>6</td>
</tr>
<tr>
<td>Bearing dynamic parameters estimation</td>
<td>3</td>
</tr>
<tr>
<td>Measurement &amp; digital processing techniques</td>
<td>2</td>
</tr>
<tr>
<td>Condition monitoring of rotating machineries</td>
<td>2</td>
</tr>
</tbody>
</table>
INTRODUCTION

Rotor Dynamics?

- Specialized branch of applied mechanics concerned with the behavior and diagnosis of rotating structures.

- It is commonly used to analyze the behavior of structures ranging from jet engines and steam turbines to auto engines and computer disk storage.

Rotor?

- Body suspended through a set of cylindrical hinges or bearings that allow it to rotate freely about an axis fixed in space.

- In this course the word “rotor” is used to describe the assembly of rotating parts in a rotating machine, including the shaft, bladed disks, impellers, bearing journals, gears, couplings, and all other elements, which are attached to the shaft.
Components of a Rotating Machinery

- Blades
- Shaft
- Rotor
- Bearing
- Foundation
- Stator
- Pedestal
Rotational motion is employed

- to achieve translation, as from the wheel to the axle
- to store energy, as in the ancient sling or modern flywheels
- to transfer power from one point to another by using belts, cogwheels, or gear trains
- to obtain kinetic energy from other kinds of energy, such as thermal, chemical, nuclear, or wind energy

Why rotational motion?

- The rotational energy has a potential for serious leaks and can easily be transformed into other forms of energy such as heat.

- In addition, in rotors there exist additional sources of energy leaks, transforming the rotor rotational energy into other forms of mechanical energy.
• On account of ever increasing demand for power and high speed transportation, the rotors of these machines are made extremely flexible, which makes the study of vibratory motion an essential part of design.

• The purpose of rotor dynamics as a subject is to keep the vibrational energy as small as possible.
Various Modes of Vibration

- Due to several factors, which contribute to the energy transfer - from rotation to other forms of motion - the rotor rotation may be accompanied by various modes of vibrations.

- Among these modes, the lateral modes of the rotor are of the greatest concern.
- Most often, they represent the lowest modes of the entire machine structure.
- Next, through the supporting bearings and through the fluid encircling the rotor (unless the rotor operates in vacuum), the rotor lateral vibrations are transmitted to the nonrotating parts of the machine.
- Eventually, the vibrations spread to the machine foundation, to adjacent equipment, building walls, and to the surrounding air in the form of acoustic waves.
Severity of the rotor vibration

Rotating machine catastrophic failure due to excessive vibrations
Bently Nevada Corporation
Forced Excitation

- There is a long list of factors which contribute to the energy transfer from rotation to these “side-effect” vibrations.
- The first and best known among them is rotor unbalance.
- When the rotor mass centerline does not coincide with its rotational axis, then mass unbalanced inertia related rotating forces occur.
- The rotor unbalance acts, therefore, in the lateral vibration mode, like an external exciting centrifugal force.
- As a result, the rotor responds with lateral vibrations with frequency, synchronous to rotational speed.
Forced Excitation

Input

(Periodic, random, ... external forces, $F$)

Rotor

Output

(Excited vibrations)
$X = X$ (time)

$F$

Time

$X$

Phase lag

Time

$F$: unbalance, torque pulsation, fluid forces, etc.
Forced Excitation

• The frequency of the rotor lateral vibrations due to unbalance will be the same as the rotational speed.
• In industry, the frequency of vibrations is usually related as ratios of the rotational speed; thus, the unbalance-related synchronous lateral vibrations are referred to as (1X) vibrations.
• If the rotor system is nonlinear, which is usually the case to a certain degree, then, in the system, more frequency components can be generated in response to an exciting force of a single frequency.
• The corresponding frequencies usually represent multiples of the excitation frequency. A nonlinear rotor synchronous (1X) response to unbalance will then be accompanied by higher harmonic components 2X, 3X, . . . .
• Additionally, often a single-frequency force can excite rotor responses with fractional 1/2X, 1/3X, . . . .
Free vibrations or transient vibrations

- “free vibrations” or “transient vibrations”, which occur when the system is excited by a short-lasting impact, causing instantaneous changes in system acceleration, velocity, and/or position.
- The system responds to the impact with free vibrations, with “natural” frequencies characteristic for the system.
Self Excited Vibration

- There exists also a third category of vibrations in physical systems, known as self-excited vibrations.
- These vibrations are steady, usually with constant amplitude, phase, and frequency.
- They are sustained by a constant source of energy, which may be external, or is a part of the system. In this type of vibrations, through the feedback mechanism, the constant energy is “portioned” by the oscillatory motion.
- The frequency of self-excited vibrations is close to one of the system natural frequencies.
- Well known are aerodynamic flutter vibrations of wings or blades, or transmission lines sustained by unidirectional wind.
Energy Feedback Transfer Mechanism in Rotating Machines

Energy Feedback Transfer Mechanism in Rotating Machines

- Internal friction
- Surrounding fluid
- Rubbing
- Anisotropy
- Electrodynamic fields...

Vibrations and changes in rotor positions
Rotating machines belong to the self excitation category. Constant supply of energy comes from rotor rotation. Whether such ‘unwanted’ self excited vibrations be tolerated?

**What Vibrations Cause in Mechanical Systems?**

As bad as all the vibrations are—from machinery efficiency standpoint—the good part is they positively also carry *information* on what caused them to occur.

However, that must be *decoded* appropriately.
How is Rotor Dynamics different from Structural Dynamics?

(i) Rotating machineries have inherent forces and moments due to dynamics of various machine elements or faults occurring in them.

(ii) Gyroscopic effects which is predominant at higher speeds, makes natural frequency speed dependent.

(iii) Bearings and seals also makes natural frequency of the rotor system speed dependent, moreover, it also makes system unstable.

(iv) The asymmetry in rotors due to operational requirements (such as keyways or slots in rotors) causes the rotor instability.

(v) The internal damping (hysteretic or friction between two mating parts in rotors) makes the system unstable.

(vi) Several other reasons for the instability due to working fluid interactions with rotor components (e.g., blades).
Critical Speed

• Speed of rotation where the amplitude of vibration is maximum.

• This amplitude is commonly excited by unbalance of the rotating structure.

• All rotating shafts, even in the absence of external load, will deflect during rotation.

• The magnitude of deflection depends upon the following:
  (a) stiffness of the shaft and its support
  (b) total mass of shaft and attached parts
  (c) unbalance of the mass with respect to the axis of rotation
  (d) the amount of damping in the system
RELATED CONCEPTS

Unbalance

Causes in case of rigid disc:
1. CG is offset from geometrical centre
2. Extra/Less mass is there at some radius from C.

Unbalance due to $1 > 2$

Rigid Disc

- If C is offset from G, then centrifugal force $= m_e \omega^2$

Unbalance $= \text{mass of rotor} \times \text{eccentricity}$

- If C and G are at same place, and additional/less mass is there at distance r, then

Centrif. force $= m_b r \omega^2$

C: Centre of rotation
G: Centre of gravity
e: eccentricity
$\omega$: spin speed
**Unbalance**

**Long rotor**

**A:** G is offset from C radially

Unbalance force = \( me \omega^2 \)

Causes:

• Some tolerances during mfg.
• During mounting disc onto the shaft
• During operation: harsh env., steam high temp. (gas turbine), wear and tear may take place

**B:** G is offset from C radially and axially

Unbalance force = \( me_r \omega^2 \)

Unbalance moment = \( (me_r \omega^2)e_z \)

Balance: 1 correction mass required

Balance: 2 correction masses required
Long rotor

C: Rotor is slightly inclined

Unbalance force = 0
Unbalance moment = \( I_d \omega^2 \psi \)

D: Rotor is slightly inclined + C & G are offset

Unbalance force = \( me \omega^2 \)
Unbalance moment = \( I_d \omega^2 \psi \)

E: Rotor is slightly inclined + C & G are offset radially and axially

Unbalance force = \( me_r \omega^2 \)
Unbalance moment = \( I_d \omega^2 \psi, (me_r \omega^2)e_z \)

Balance: in 2 different planes (Dynamic balancing)

Unbalance force will be transmitted to bearings & hence is an undesirable form
RELATED CONCEPTS

Whirling Transverse vibration which takes place when the rotor is flexible

(a) A simply supported shaft with a disc at the from mid-span
(b) A simply supported shaft with a disc away mid-span
(c) A cantilever shaft with a point disc at the free end
(d) A cantilever shaft with a rigid disc at the free end

The spinning and precession (whirling) motions of the disc
Whirling

- Flexible rotor: Shaft will deflect/deform
- Combination of: (i) Spinning of rotor about its own axis
  (ii) Rotation of shaft about the bearing centre (Transverse vib.)
- Whirling frequency \( \nu \neq \omega \) (in general)
- If whirling is due to unbalance, it is expected \( \nu = \omega \): Synchronous whirl cond.

- **Forward whirl**
  \[ \nu = \omega \]

- **Backward whirl**
  \[ \nu = -\omega \]

- No particular pattern
- This takes place due to GE or due to bearing because bearing has a property that changes with speed
RELATED CONCEPTS

Wobbling

Takes place when disc is offset from the centre

(a) A simply supported shaft with a disc near the bearing
(b) A cantilever shaft with a rigid disc at the free end

Wobbling of the disc in a rotor system
TERMINOLOGIES

Campbell Diagram

• Also known as "Whirl Speed Map" or a "Frequency Interference Diagram"
• Simple rotor system.
• Pink and blue curves show the backward whirl (BW) and forward whirl (FW) modes, which diverge as the spin speed increases.
• BW frequency or the FW frequency equal the spin speed $\Omega$, indicated by the intersections A and B with the synchronous spin speed line, the response of the rotor may show a peak. This is called a critical speed.
## SUMMARY OF VARIOUS ROTOR DYNAMIC PHENOMENA

<table>
<thead>
<tr>
<th>S. N.</th>
<th>Phenomena</th>
<th>Caused by</th>
<th>Reported/Interpreted by</th>
<th>Remarks (Theoretical/Experimental)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Whirling</td>
<td>Unbalance</td>
<td>Rankine (1869)</td>
<td>General motion (Th)</td>
</tr>
<tr>
<td>2</td>
<td>Self centering of rotor</td>
<td>Unbalance</td>
<td>De Laval (1983)</td>
<td>Unbalance response (Exp)</td>
</tr>
<tr>
<td>3</td>
<td>Synchronous whirling</td>
<td>Unbalance</td>
<td>Föppl (1895)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Critical speed</td>
<td>Unbalance</td>
<td>Dunkerley (1895)</td>
<td>Resonance (Th)</td>
</tr>
<tr>
<td>5</td>
<td>Second critical speed</td>
<td>Unbalance</td>
<td>Kerr (1916)</td>
<td>Resonance (Exp)</td>
</tr>
<tr>
<td>6</td>
<td>Stable supercritical response</td>
<td>Unbalance</td>
<td>Jeffcott (1919)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Secondary resonance</td>
<td>Gravity</td>
<td>Stodola (1924)</td>
<td>Unbalance response (Th)</td>
</tr>
<tr>
<td>8</td>
<td>Instability</td>
<td>Shaft asymmetry</td>
<td>Prandtl (1918)</td>
<td>Instability analysis (Th)</td>
</tr>
<tr>
<td>9</td>
<td>Gyroscopic effect</td>
<td>Rotor wobbling</td>
<td>Stodola (1924)</td>
<td>Free vibrations (Th)</td>
</tr>
<tr>
<td>10</td>
<td>Threshold spin speed for instability</td>
<td>Internal damping</td>
<td>Newkirk (1924), Kimball (1924), Smith (1933), Crandall (1961)</td>
<td>Instability analysis (Th)</td>
</tr>
<tr>
<td>11</td>
<td>Threshold spin speed for instability</td>
<td>Dissymmetry of bearing stiffness</td>
<td>Smith (1933)</td>
<td>(1933)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>12</td>
<td>Oil whip</td>
<td>Nonlinear action of the oil wedge in a journal bearing</td>
<td>Newkirk and Taylor (1925)</td>
<td>Instability analysis (Th)</td>
</tr>
<tr>
<td>13</td>
<td>Self-excite vibration</td>
<td>Contact between rotor and stator</td>
<td>Baker (1933)</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Oil whip</td>
<td>Hydrodynamic bearing</td>
<td>Hori (1959)</td>
<td>(Th)</td>
</tr>
<tr>
<td>15</td>
<td>Steam whirl</td>
<td>Steam injection on turbine blades</td>
<td>Thomas (1958)</td>
<td>(Th)</td>
</tr>
<tr>
<td>16</td>
<td>Flow induced vibrations</td>
<td>Hollow rotor containing fluid</td>
<td>Kollmann (1962), Ehrich (1965), Kuipers (1964), Wolf (1968)</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Instability</td>
<td>Seals</td>
<td>Jenny (1980)</td>
<td>(Exp)</td>
</tr>
<tr>
<td>18</td>
<td>Subharmonics/</td>
<td>Combination resonance</td>
<td>Nonlinearity (ball bearing)</td>
<td>Yamamoto (1955, 1957)</td>
</tr>
<tr>
<td>19</td>
<td>Nonlinear resonance</td>
<td>Oil films in journal bearings</td>
<td>Tondl (1965)</td>
<td>(Th)</td>
</tr>
<tr>
<td>20</td>
<td>Subhamonic resonances</td>
<td>Squeeze film dampers</td>
<td>Ehrich (1966)</td>
<td>(Exp)</td>
</tr>
<tr>
<td>21</td>
<td>Nonstationary response</td>
<td>Constant/variable accelerations of rotor</td>
<td>Lewis (1932)</td>
<td>(Th)</td>
</tr>
<tr>
<td>22</td>
<td>Shaft vibrations at critical speeds</td>
<td>Varying spin speeds</td>
<td>Natanzon (1952)</td>
<td>(Th)</td>
</tr>
<tr>
<td>23</td>
<td>Shaft general motion</td>
<td>Varying spin speeds</td>
<td>Grobov (1953, 1955)</td>
<td>(Th)</td>
</tr>
<tr>
<td>24</td>
<td>Damped critical speeds</td>
<td>Hydrodynamic bearings</td>
<td>Ruhl and Booker’s (1972), and Lund’s (1974)</td>
<td>FEM (Th)</td>
</tr>
</tbody>
</table>
**HISTORICAL DEVELOPMENT OF THE FIELD**

Early history is replete with the interplay of theory and practice.

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
</table>
| 1869 | W. J. M. Rankine: First analysis of a spinning shaft.
| 1889 | Gustaf de Laval, a Swedish engineer, ran a steam turbine to supercritical speeds.
| 1895 | Dunkerley published an exp. paper describing supercritical speeds.
| 1916 | Kerr published a paper showing experimental evidence of a second critical speed.
| 1919 | Henry Jeffcott published a paper now considered classic in the *Philosophical Magazine* in 1919 in which he confirmed the existence of stable supercritical speeds.

**August Föppl** published much the same conclusions in 1895, but history largely ignored his work.
Jeffcott used an undamped model to show that an unbalanced disc would whirl synchronously with:
• Heavy side flying out when the rotation was subcritical
• Heavy side flying in when the rotation was supercritical

Also the behaviour of Laval rotors at high speed was confirmed by his theory.
Jeffcott rotor: flexible shaft of negligible mass with a rigid disc at the midspan.

The bearings are rigidly supported, and viscous damping acts to oppose absolute motion of the disc.

This simplified model is also called the Laval rotor, named after de Laval.
Between the work of Jeffcott and the start of World War II there was much work in the area of instabilities and modeling techniques culminating in the work of Nils Otto Myklestad and M. A. Prohl which led to the transfer matrix method (TMM) for analyzing rotors. The most prevalent method used today for rotordynamics analysis is the finite element method.