1. Fluid Film Lubrication

In fluid film lubrication regime the sliding surfaces are completely separated by a film of liquid or gaseous lubricant. Fig. 1(a) highlights few possible methods to achieve the reduction in normal and shear stresses. The first two concepts are useful for few degree oscillations. Continuous relative motion such as rotational motion requires solid lubricants, boundary lubricants, rolling elements and associate lubricants, pressurized (hydrostatic, aerostatic, hydrodynamic, aerodynamic) lubrication or magnetic/electric field.

Fig. 1(a): Various concepts to separate two solid surfaces

1.1 Classification:

- **Hydrodynamic (Aerodynamic):**
  - Converging wedge shaped geometry; as shown in Fig. 1(b) is essential for this lubrication.
  - Viscosity of lubricant plays an important role to support the load.

Fig. 1(b): Converging wedge shape geometry

- **Squeeze Film:**
  - Load and/or speed variation generate squeeze film action.
- Viscosity of lubricant plays important role.

Fig. 2: Squeeze lubrication.

- **Hydrostatic (Aerostatic):**
  - External pressure of fluid needs to be supplied to generate hydrostatic/aerostatic lubrication.
  - Independent or to support hydrodynamic lubrication.

Fig. 3: Hydrostatic

Every mechanism of fluid film lubrication is suitable for a particular set of operating and environmental condition. For example, hydrostatic lubrication that separates two surfaces by an external pressure source. It is suitable for extremely high load carrying capacity at low speed or at highly controlled precision works. This mechanism finds its application in large telescopes, radar tracking units, machine tools and gyroscopes. In reverse, aerostatic (here aero is not restricted to air, but nitrogen and helium are also used) works well in high speed (from 25,000 rpm to 7,00,000 rpm) and light load applications even in all odd environment temperature conditions (-2000°C to 20000°C). As pressure is generated and supplied by external sources, it is one of the expensive approaches to separate two surfaces. Further, if applied load is reduced, the film thickness (separation between tribo-pair) will increase. Similarly if more load is added to the moving surface, the film thickness will decrease. If a load greater than design load is applied the tribo-surface will not be able separated. To reduce this sensitivity feed back control system is used, which increases the cost of overall system. To compensate cost, often a hybrid concept of hydrodynamic + hydrostatic or aerodynamic + aerostatic is used to achieve best of both the mechanisms of fluid film lubrications. Aerodynamic tribo-pair requires very sophisticated manufacturing facilities, and can sustain 5-10 % load compared to tribo-pair based on hydrodynamic lubrication.
In a hydrodynamic lubrication mechanism, a fluid is drawn into the region between the relatively-moving surface by the virtue of its viscosity and adhesion to the surfaces; and due to the converging geometry of the surfaces (as shown in Fig. 1(b)) a pressure is generated within the fluid that separates the tribo-surfaces. The separating film is only generated when there is relative motion. Higher the relative velocity thicker is the lubricant layer. This lubrication mechanism is referred as “the ideal form of lubrication”, since solid surfaces are prevented from coming into contact (as shown in Fig. 4(a), where two surfaces are completely separated) that provides high resistance to wear and low friction. Lubricant viscosity plays a very important role in the hydrodynamic lubrication. Minimum film thickness ($h_{\text{min}}$), which is a critical design criterion, is a function of relative velocity ($U$) and applied load ($W$). The expression $h_{\text{min}} \propto (U/W)^{1/2}$ generally guides preliminary design of fluid film bearings. Force is calculated using the expression $F = (\eta U/h)A$, where $\eta$ is lubricant dynamic viscosity, $U$ is relative velocity, and $h$ is film thickness. This expression generally misguides and indicates thicker the lubricant film lesser the friction. In reality, friction force in hydrodynamic lubrication is represented as:

$$F = \int_{A} r \, dA = \int \left( \frac{\eta U}{h} + \frac{h}{2R} \frac{\partial P}{\partial \theta} \right) \, dA$$

This expression shows friction depends on pressure generation as well as velocity. For example in journal bearing, friction loss can be represented as:

$$F = \frac{\eta ULR \pi}{C \sqrt{1-e^2}} \left( 2 + e \right) \left( 1+e \right) + \frac{C \phi W_{\phi}}{2R}$$

In this expression, $W_{\phi}$ is load perpendicular to load line. This component of load decreases with increase in eccentricity ratio. Best way to judge the friction performance is evaluate the coefficient of friction ($f$). For example, for a typical journal bearing operation, at eccentricity ratio 0.5, 0.6, 0.7, 0.8 and 0.9, friction force is: 36.5 N, 39N, 43 N, 52N, 74 N respectively; while load capacity is: 4500 N, 8000 N, 12200N, 23540 N, and 56994 N respectively. In this way coefficient of friction will be 0.008, 0.0048, 0.0035, 0.0022, and 0.0013 respectively. Therefore one can say thinner the hydrodynamic film, better is the friction performance.

In addition to having low frictional drag and almost nil wear rate, hydrodynamic mechanism is self acting and requires little attention. However, hydrodynamic action cannot be sustained at low design speed. Further it breaks down during starting, direction changing and stopping.
Another well-known mechanism that always occurs at the start/stop of tribo-pair is boundary lubrication. This lubrication mechanism assumes (as shown in Fig. 4(c)) almost negligible separation between tribo-surfaces that occurs at high load and very low speed. In such cases, the working fluid between tribo-pairs adheres to or "wets" the surfaces, and carries a fraction of imposed load. The physical and chemical properties of thin molecular films and the surfaces to which they are attached determine the overall tribological behavior. The friction and wear, in the case of boundary lubrication, are much higher compared to those in full film lubrication. That is why hybrid concept of hydrostatic operation at start and hydrodynamic operation at running conditions came into existence. Third important fluid film lubrication mechanism is mixed lubrication which assumes (as shown in Fig. 4(d)) solid contact between some asperities of tribo-pairs, while rest of the area covered by lubricant. The lubrication mechanism in this regime is governed by a combination of boundary and hydrodynamic lubrication. Interaction takes place between one or more molecular layers of boundary lubricating film.

Lowest friction producing fluid film lubrication mechanism is Elasto-Hydrodynamic Lubrication (EHL). EHL mostly occurs in rolling element bearings, gears, cam-follower contact. As the name suggests this lubrication mechanism utilizes: (1) elastic deformation, and (2) hydrodynamics. Relative velocity between tribo-pair develops hydrodynamic action. Excessive load or relatively soft surface results in elastic deformation of surface(s). In case of excessive load, generated fluid pressure is very high (ranging between 0.5 to 2 GPa) and the surfaces deform elastically. As lubricant viscosity is a strong function of pressure, particularly at high pressure, lubricant-viscosity may rise (as much as 10 orders of magnitude) considerably, and this further assists the formation of an effective lubrication. EHL analysis involves an iterative procedure to establish compatibility between hydrodynamic pressure developed by relative motion and separation between tribo-pair caused by this pressure. Simplest way to analyze EHL is by making assumption of film thickness, solve using hydrodynamic equations, evaluate elastic deformation of surfaces, modify film thickness and iterate. The iteration continues until the modified film thickness distribution matches with the new film thickness distribution. One of the notable points of EHL compared to hydrodynamic lubrication is the negligible effect of load on minimum film thickness $h_{\text{min}} \propto W^{0.075}$ and significant effect of relative velocity on film thickness $h_{\text{min}} \propto U^{0.68}$. 

Fig. 4(c): Boundary lubrication, (d): Mixed lubrication.
2. Reynolds Equation

Fig. 5(a): Fluid film lubrication between two plates, (b) Velocities of Plates

- In 1886, Reynolds derived an equation for estimation of pressure distribution for “Fluid Film Lubrication”.

- Quantification of fluid film lubrication can be made by solving Reynolds’ Equation, which provides fluid film pressure as a function of coordinates and time. Reynolds equation helps to predict hydrodynamic, squeeze, and hydrostatic film mechanisms.

For liquid lubricant under isothermal conditions

\[
\frac{\partial}{\partial x} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial z} \right) = 6 \left\{ \frac{\partial}{\partial x} \left( U_1 + U_2 \right) h + \frac{\partial}{\partial z} \left( U_1 + U_2 \right) h + 2 \left[ \left( w_h - U_1 \frac{\partial h}{\partial x} - U_2 \frac{\partial h}{\partial z} \right) - w_0 \right] \right\}
\]

...Eq.(1)

for geometry shown in Fig. 5.

for hydrostatic case this equation will be reduced to:

\[
\frac{\partial}{\partial x} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial z} \right) = 0
\]

...Eq.(2)

for hydrodynamic case this equation will be reduced to:

\[
\frac{\partial}{\partial x} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial z} \right) = 6 \left\{ \frac{\partial}{\partial x} \left( U_2 - U_1 \right) h + \frac{\partial}{\partial z} \left( W_2 - W_1 \right) h \right\}
\]

...Eq.(5.3)

for squeeze film case this equation will be reduced to:

\[
\frac{\partial}{\partial x} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial z} \right) = 12 \left( V_h - V_0 \right)
\]

...Eq.(5.4)

2.1 Derivation of Reynolds’ Equation:

To model the pressure as a function of angle of inclination, let us consider a fluid element subjected to pressure and viscous forces, assuming gravity and inertia forces(as shown in Fig. 6) acting on fluid element to be negligible.
2.1.1 Conservation of Momentum: Balancing forces. For element shown in Fig. 6,

\[
p dy dz + \left( \tau + \frac{\partial \tau}{\partial y} \frac{dy}{dz} \right) dx dz = \left( p + \frac{\partial p}{\partial x} \right) dy dz + \tau dx dz
\]

\[
\left( \frac{\partial \tau}{\partial y} - \frac{\partial p}{\partial x} \right) dx dy dz = 0
\]

\[
\Rightarrow \frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}
\]

Eq. (5)

For laminar flow of Newtonian fluid,

\[
\tau = \eta \frac{\partial u}{\partial y}
\]

\[
\frac{\partial}{\partial y} \left( \eta \frac{\partial u}{\partial y} \right) = \frac{\partial p}{\partial x}
\]

Eq. (6)

Equation (6) is based on following assumptions:
1. Negligible inertia terms
2. Negligible pressure gradient in the direction of film thickness
3. Newtonian fluid
Assuming constant value of viscosity, \( \frac{\partial P}{\partial x} = \eta \frac{\partial^2 u}{\partial y^2} \) \( \text{....Eq.7) } \)

Similarly on force balance in z direction, \( \frac{\partial P}{\partial z} = \eta \frac{\partial^2 w}{\partial y^2} \) \( \text{....Eq.8) } \)

To find flow velocity \( u \) in \( x \) - direction,

integrate \( \frac{\partial P}{\partial x} = \eta \frac{\partial^2 u}{\partial y^2} \) two times.

On integrating first time \( \eta \frac{\partial u}{\partial y} = \frac{\partial P}{\partial x} y + C_1 \) \( \text{....Eq.9) } \)

On integrating second time \( \eta u = \frac{\partial P}{\partial x} \frac{y^2}{2} + C_1 y + C_2 \) \( \text{....Eq.10) } \)

Assuming no slip at liquid-solid boundary;

\( y = 0, u = U_2, y = h, u = U_1 \)

Utilizing these boundary conditions, values of integration constants can be evaluated.

\[ \eta U_2 = \frac{\partial P}{\partial x} \frac{0^2}{2} + C_1 0 + C_2 \Rightarrow \eta U_2 = C_2 \]

\[ \eta U_1 = \frac{\partial P}{\partial x} \frac{h^2}{2} + C_1 h + C_2, \frac{\eta(U_1-U_2)}{h} - \frac{\partial P}{\partial x} \frac{h}{2} = C_1 \]

On substituting \( C_1 \) and \( C_2 \):

\[ u = \left( \frac{y^2 - yh}{2\eta} \right) \frac{\partial P}{\partial x} + (U_1-U_2) \frac{y}{h} + U_2 \] \( \text{....Eq.11) } \)

Equation (11) is applicable for following assumptions:

1. Negligible inertia terms.
2. Negligible pressure gradient in the direction of film thickness.
4. Constant value of viscosity.
5. No slip at liquid solid boundary.

In Eq.(11) on right hand side there are three terms, two velocity terms and one pressure term. Velocity terms represent “shear flow” also known as “Couette flow”. Flow due to pressure gradient is termed as “Poiseuille flow”. Similarly flow velocity in \( z \) dir.

\[ w = \left( \frac{y^2 - yh}{2\eta} \right) \frac{\partial P}{\partial z} + (W_1-W_z) \frac{y}{h} + W_z \] \( \text{....Eq.12) } \)
The Poiseuille flow term retards the fluid flow at entrance as shown in Fig. 7. This pressure term boasts flow at exit as indicated in Fig. 7. In this figure \( U_a = U_1 - U_2 \).

### 2.1.2 Conservation of Mass: Continuity Equation

To derive the Reynolds equation, Eqs. (11 & 12) must be coupled with continuity equation. Continuity equation for incompressible fluid can be expressed as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

...Eq. (13)

![Fig. 9: Inclined plates with fluid element and coordinate system](image)

Integrating this equation in the y-direction from \( y = 0 \) to \( y = h \).

\[
u = \left( \frac{y^2 - yh}{2\eta} \right) \frac{\partial P}{\partial x} + (U_1 - U_2) \frac{y}{h} + U_2
\]

Eq. (14)

\[
w = \left( \frac{y^2 - yh}{2\eta} \right) \frac{\partial P}{\partial z} + (W_1 - W_2) \frac{y}{h} + W_2
\]

Eq. (15)

\[
\int_0^h \frac{\partial u}{\partial x} dy + \int_0^h dv + \int_0^h \frac{\partial w}{\partial z} dy = 0
\]

Eq. (16)

Using Leibnitz rule

\[
\int_a^b \frac{\partial u(y,x)}{\partial x} dy = \frac{d}{dx} \int_a^b u(y) dy - u(b,x) \frac{db}{dx} + u(a,x) \frac{da}{dx}
\]

Using

\[
\int_0^h \frac{\partial u(x,y)}{\partial x} dy = \frac{\partial}{\partial x} \int_0^h u(x,y) dy - u(x,h) \frac{\partial h}{\partial x}
\]

\[
\Rightarrow \int_0^h \frac{\partial u}{\partial x} dy = \frac{\partial}{\partial x} \int_0^h u dy - (U_1) \frac{\partial h}{\partial x}
\]

Eq. (17)
Using Eq. (14)

\[ u = \left( \frac{y^3 - yh}{2\eta} \right) \frac{\partial P}{\partial x} + \frac{(U_1 - U_2)}{h} y + U_2 \]

\[ \int_0^h u \, dy = \frac{1}{2\eta} \left[ \frac{y^3}{3} - \frac{y^2 h}{2} \right]_0^h \frac{\partial P}{\partial x} + \frac{(U_1 - U_2)}{h} \left[ \frac{y^2}{2} \right]_0^h + U_2 h \]

\[ = \frac{1}{2\eta} \left[ -\frac{h^3}{6} \frac{\partial P}{\partial x} + \frac{(U_1 - U_2)}{h} \frac{h^2}{2} + U_2 h \right] \]

\[ = -\frac{h^3}{12\eta} \frac{\partial P}{\partial x} + \frac{h}{2} (U_1 + U_2) + U_2 h \]

\[ = -\frac{h^3}{12\eta} \frac{\partial P}{\partial x} + \frac{h}{2} (U_1 + U_2) \] \[ \ldots \text{Eq. (18)} \]

In substituting Equation (17)

\[ \int_0^h \frac{\partial u}{\partial x} \, dy = \frac{\partial}{\partial x} \left[ -\frac{h^3}{12\eta} \frac{\partial P}{\partial x} + \frac{h}{2} (U_1 + U_2) \right] - (U_1) \frac{\partial h}{\partial x} \] \[ \ldots \text{Eq. (19)} \]

\[ \int_0^h \frac{\partial u}{\partial x} \, dy = \frac{\partial}{\partial x} \left[ -\frac{h^3}{12\eta} \frac{\partial P}{\partial x} + \frac{h}{2} (U_1 + U_2) \right] - (U_1) \frac{\partial h}{\partial x} \]

\[ \int_0^h \frac{\partial u}{\partial x} \, dy = \frac{\partial}{\partial x} \left[ -\frac{h^3}{12\eta} \frac{\partial P}{\partial x} + \frac{h}{2} (U_1 + U_2 - 2U_1) \right] \]

\[ \int_0^h \frac{\partial u}{\partial x} \, dy = \frac{\partial}{\partial x} \left[ -\frac{h^3}{12\eta} \frac{\partial P}{\partial x} + \frac{h}{2} (U_2 - U_1) \right] \] \[ \ldots \text{Eq. (5.20)} \]

Similarly in z-direction

\[ \int_0^h \frac{\partial w}{\partial z} \, dy = -\frac{\partial}{\partial z} \left( \frac{h^3}{12\eta} \frac{\partial P}{\partial z} - \frac{1}{2} \frac{\partial (W_1 - W_2)}{\partial z} h \right) \] \[ \ldots \text{Eq. (5.21)} \]

Combining Equations (5.16), (5.20) & (5.21):
Equation (5.22) is applicable for following assumptions:

1. Negligible inertia terms
2. Negligible pressure gradient in the direction of film thickness
3. Newtonian fluid
4. Constant value of viscosity
5. No slip at liquid solid boundary
6. Neglecting angle of inclination for coordinate system
7. Incompressible flow

2.2 Reynolds’ Equation for Incompressible Lubricants:

\[
- \frac{\partial}{\partial x} \left[ \frac{h^3}{12\eta} \frac{\partial P}{\partial x} \right] - \frac{1}{2} \frac{\partial}{\partial x} \left[ \left( U_1 - U_2 \right) h \right] + \left( V_h - V_0 \right) h = 0
\]

Or

\[
\frac{\partial}{\partial x} \left[ \frac{h^3}{12\eta} \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{h^3}{12\eta} \frac{\partial P}{\partial z} \right] = 0
\]

\[
= \frac{1}{2} \frac{\partial}{\partial x} \left[ \left( U_2 - U_1 \right) h \right] + \left( V_h - V_0 \right) + \frac{1}{2} \frac{\partial}{\partial z} \left[ \left( W_2 - W_1 \right) h \right]
\]

Eq.(22)

- Left hand side term is called pressure term.
- Right hand side terms are called source terms.
- \( \partial U/\partial x, \partial W/\partial z \) : Called stretching action.
- \( \partial h/\partial x, \partial h/\partial z \) : Wedge action(inclined surfaces).
- \( (V_1 - V_2) \) : Squeeze action(bearing surfaces move perpendicular to each other).
If surface is very soft and stretchable, then velocity gradient on the surface of bearing (as shown in Fig. 11) occurs.

Reynolds equation (Eq. 22) can be simplified by making few assumptions.

Assuming constant viscosity(\(\eta\))

\[
\frac{\partial}{\partial x} \left( h^3 \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial z} \left( h^3 \frac{\partial P}{\partial z} \right) = 6\eta \frac{\partial(U_2 - U_1)h}{\partial x} + 12\eta \frac{\partial h}{\partial z} + 6\eta \frac{\partial(W_2 - W_1)h}{\partial z}
\]

Assuming no relative velocity in Z-direction (\(W_1 = W_2 = 0\)).

\[
\frac{\partial}{\partial x} \left( h^3 \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial z} \left( h^3 \frac{\partial P}{\partial z} \right) = 6\eta \frac{\partial(U_2 - U_1)h}{\partial x} + 12\eta \frac{\partial h}{\partial z}
\]

Eq.(23)

Equation (23) is applicable for following assumptions:
1. Negligible inertia terms
2. Negligible pressure gradient in the direction of film thickness
3. Newtonian fluid
4. Constant value of viscosity
5. No slip at liquid solid boundary
6. Neglecting angle of inclination for coordinate system
7. Incompressible flow
8. Relative tangential velocity only in x-dir

Assuming no stretching action.

\[ \frac{\partial}{\partial x} \left( h^3 \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial z} \left( h^3 \frac{\partial P}{\partial z} \right) = 6\eta \left( U_2 - U_1 \right) \frac{\partial h}{\partial x} + 2 \frac{\partial h}{\partial t} \]

Assuming one surface with zero tangential velocity.

\[ \frac{\partial}{\partial x} \left( h^3 \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial z} \left( h^3 \frac{\partial P}{\partial z} \right) = 6\eta \left( U_2 \frac{\partial h}{\partial x} + 2 \frac{\partial h}{\partial t} \right) \]

Eq. (24)

Equation (24) is applicable for following assumptions:

1. Negligible inertia terms
2. Negligible pressure gradient in the direction of film thickness
3. Newtonian fluid
4. Constant value of viscosity
5. No slip at liquid solid boundary
6. Neglecting angle of inclination for coordinate system
7. Incompressible flow
8. Relative tangential velocity only in x-direction
9. Both rigid surfaces
10. Only inclined surface slides

NOTE: Please refer to Lectures 19 and 20 of Video Course on Tribology for detailed explanation of the contents.

Source:
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